An error correction. Letter to the editor

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Dear Editor:

In 2004 the JSP published an article [Toom] authored by one of us, which studied a certain random process with discrete time. Components were enumerated by integer numbers and every component had only two possible states denoted $\ominus$ and $\oplus$ and called *minus* and *plus*. Thus the configuration space was $\{\ominus,\ \oplus\}^{\mathbb{Z}}$. We denote by $\mathcal{M}$ the set of translation-invariant normalized measures on this space and by $\delta_\ominus,\ \delta_\oplus \in \mathcal{M}$ the measures concentrated in the configurations "all minuses" and "all pluses" respectively.

The initial condition was $\delta_\ominus$. At every step of the discrete time two operators acted. The first of them, called *flip*, was denoted Flip_β; under its action any minus turned into plus with probability β independently of states and fate of other components. The other operator, called *annihilation* was denoted by Ann_α. Under its action, whenever a plus was a left neighbor of a minus, either both of them disappeared with a probability α, or both remained intact with a probability $1-\alpha$ independently of states of all the other components. Following [Toom], we write operators on the right side of measures on which they act and denote by μ_t the

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result of t application of our two operators, first Flip_β, then Ann_α to the initial condition:

$$\mu_t = \delta_\ominus (\mathrm{Flip}_\beta \mathrm{Ann}_\alpha)^t. \tag{1}$$

The main result of [Toom] was this:

$$\left.\begin{array}{l}\text{If } 0 < \alpha < 1 \text{, then for all natural } t \text{ the frequency of} \\ \text{pluses in the measure } \mu_t \text{ does not exceed } 300 \cdot \beta/\alpha^2 .\end{array}\right\} \tag{2}$$

The purpose of this letter is to state the following:

1) The article [Toom] contains an error, but all the main results of [Toom] (stated as theorems there) are still true.

2) Correction of this error rather improves than deteriorates the results; in fact, it allows us to substitute 250 instead of 300 in (2) .

3) The restriction $\alpha < 1$ assumed throughout [Toom] is unnecessary and all the theorems of [Toom] are true for the case $\alpha = 1$ also. In fact, for this case we obtain numerical estimations (presented below) which are better than those obtained for $\alpha < 1$.

Now let us explain our statements. We use the same enumeration of formulas as in [Toom]. The error of [Toom] was the unnumbered affirmation (right after the formula (40)) that the quantities defined in

(38) satisfy the initial condiction

$$S_1(1) = 1/q, \quad S_2(1) = S_3(1) = S_4(1) = 0,$$

while in fact

$$S_1(1) = q, \quad S_2(1) = S_3(1) = S_4(1) = 0.$$

We use the same values of parameters p and q as those given in (39) in [Toom]. Thus $q < 1$ and the correction assigns a smaller value to $S_1(1)$. After that, following essentially the same way as in [Toom], we obtain the better estimation.

Now let us prove that all the theorems of [Toom] are true for $\alpha = 1$. It is sufficient to prove that the process μ_t is defined when $\alpha = 1$. Let us denote by μ_{chess} the (unique) measure in $\mathcal{M}$ defined by the condition

$$\mu_{chess}(\ominus,\ \oplus) = \mu_{chess}(\oplus,\ \ominus) = 1/2. \tag{3}$$

The operator Ann_1 cannot be applied to μ_{chess}, which was the reason why [Toom] excluded the case $\alpha = 1$. However, Ann_1 can be applied to all the other measures in $\mathcal{M}$. Thus, to include the case $\alpha = 1$, it is sufficient to prove that we never have to apply Ann_α to μ_{chess} in the course of inductive generation of measures μ_t. According to (1) , Ann_α is always applied after Flip_β. It is evident that

$$\mu(\oplus,\ \oplus) \geq \beta^2 \tag{4}$$

for any measure μ, which is a result of application of operator Flip_β. We may exclude the trivial case $\beta = 0$. Then the right side of (4) is positive, whence the left side is positive, which is incompatible with the conditions (3) .

Finally, here are some estimations in the case $\alpha = 1$, tighter than in the case $\alpha < 1$:

1) If $\alpha = 1$, then for all natural t the frequency of $\oplus$ in the measure μ_t does not exceed $150 \cdot \beta$.

2) If $\alpha = 1$ and $\beta \geq 0.36$, the measure μ_t tends to $\delta_\oplus$ when $t \to \infty$.

The technical details of our arguments may be found in [Ramos].

References

[Ramos] A. D. Ramos. Particle processes with variable length. Ph. D. thesis (in portuguese). Available at: `http//www.de.ufpe.br/~toom/ensino/doutorado/alunos/index.htm`

[Toom] A. Toom, Non-ergodicity in a 1-D particle process with variable length, *Journal of Statistical Physics* 115 (2004) 895-924.