

We consider d -dimensional random processes with linear local interaction, whose components are real random variables a_s^t . Following Hammersley, we call them *harnesses*. Here $s \in \mathbb{Z}^d$ and $t = 0, 1, 2, \dots$. Choose vectors $v_1, \dots, v_N \in \mathbb{Z}^d$, whose differences generate \mathbb{Z}^d , and positive numbers w_1, \dots, w_N , whose sum equals 1. The process is defined by

$$a_s^t = \sum_{i=1}^N w_i \cdot a_{s+v_i}^{t-1} + \nu_s^t \quad \text{for all } t = 1, 2, 3, \dots, \quad \text{where } a_s^0 \equiv 0,$$

where ν_s^t are i.i.d. random variables, distributed as some symmetric random variable ν , called *noise*.

For any $\sigma \in \mathbb{Z}_+^d$ define $\Delta_\sigma a_s^t$ as follows. If $\sigma = (0, \dots, 0)$, $\Delta_\sigma a_s^t = a_s^t$. Then by induction: $\Delta_{\sigma+e_i} a_s^t = \Delta_\sigma a_{s+e_i}^t - \Delta_\sigma a_s^t$, where e_i is the d -dimensional vector, whose i -th component is one and other components are zeroes. Denote $|\sigma|$ the sum of components of σ .

For any symmetric random variable ξ its P-decay is defined as the supremum of those r , for which the r -th absolute moment of ξ is finite. Convergence a.s., in probability and in law when $t \rightarrow \infty$ is examined:

If $d = 1$, $\sigma = 0$ or $d = 2$, $\sigma = (0, 0)$, $\Delta_\sigma a_s^t$ diverges. In all the other cases:

If P-decay $(\nu) < (d+2)/(d+|\sigma|)$, $\Delta_\sigma a_s^t$ diverges.

If P-decay $(\nu) > (d+2)/(d+|\sigma|)$, $\Delta_\sigma a_s^t$ converges and P-decay $(\lim \Delta_\sigma a_s^t) = \text{P-decay}(\nu)$.

For any symmetric random variable ξ its E-decay is defined as $\lim_{x \rightarrow \infty} \inf \log_x(-\ln \text{Prob}(\xi > x))$. Let E-decay $(\nu) > 0$. Exclude the two divergent cases $d = 1$, $\sigma = 0$ and $d = 2$, $\sigma = (0, 0)$. In all the other cases:

If $d > 2$, E-decay $(\lim a_s^t) = \min(\text{E-decay}(\nu), (d+2)/2)$.

If $|\sigma| = 1$, E-decay $(\lim \Delta_\sigma a_s^t) = \min(\text{E-decay}(\nu), d+2)$.

If $|\sigma| \geq 2$, E-decay $(\lim \Delta_\sigma a_s^t) = \text{E-decay}(\nu)$. (Received September 21, 1996)