Wars in American mathematical education

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I am a mathematician interested in mathematical education. I have done research and taught for two decades in Russia, a decade in USA and a couple of years in Brazil. I left Russia forever not without reason. Russia had and still has some ugly features with which I don't want to have anything common. However, Russian public school mathematical education is better than American and Brazilian and it has been so for several decades including the Communist period. This sounds like a paradox, but you will see that it is more than just my personal impression. Why this is so? This article contributes to the answer.

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Several years ago the Department of Education of USA, helped by analogous offices of many other countries, conducted the Third International Mathematics and Science Study (TIMSS). [1] Its purpose was to compare the average quality of mathematics and science education at several levels in as many countries as possible. The most shocking result was the relative decline of American students while attending schools: American 4-th graders scored above the world average, 8-th graders were a little below average and 12-th graders were much below average. It is difficult to avoid a conclusion that American children come to school well prepared, but American schools are worse that schools of many other countires. In the 8-th grade (where the number of participants was the greatest) the first several places went to East-Asian countries. Next several places went to European countries and Russia was among them: a little worse than France, but a little better than England and Germany. This is a very good result for a country where most people were illiterate in the beginning of the 20-th century. USA was significantly below these groups. The only Latino-American country mentioned there is Colombia; it is almost at the very bottom of the list. I have no doubt that if Brazil participated, its result would be also very poor.

[1] About TIMSS see web site **http://timss.bc.edu**.

The disturbing results of TIMSS did not remain unnoticed in USA. On June 9, 1997 Senator Robert Byrd started his speech as follows:

> Mr. President, over the past decade, I have been continually puzzled by our Nation's failure to produce better students despite public concern and despite the billions of Federal dollars which annually are appropriated for various programs intended to aid and improve education. $< \cdots >$ Particularly in mathematics, where our kids will have to be especially skilled, the United States ranks 28th in average mathematics performance according to a study of 8th graders published in 1996. Japan ranked third. [2]

I think that the real situation in American mathematical education is even worse than TIMSS shows because TIMSS followed the anti-theoretical bias of American educators. By "theory" I don't mean anything too advanced. Let me give an example. When I was in high school, we studied the quadratic function in a rather complete manner. In particular it was obligatory for all Russian schools to teach children to derive the formula for roots of a quadratic equation $ax^2 + bx + c = 0$, to prove that the sum of roots equals $-b/a$ and that their product equals c/a and to use these facts to factorize the trinomial. Most American students whom I ever met were not even aware of most of these facts. Other immigrants from Russia have similar impressions. Gregory Galperin who always was interested not only in research, but also in teaching, wrote to me soon after starting to teach in America:

> I am very surprised that all (!) the American students know (on some particular examples) how to factor a given quadratic polynomial but do not understand that the numbers inside the brackets are the roots of the equation as well as they do not even suspect what the sum and the product of the roots are equal to. E.g., my graduate students in differential geometry could calculate quite hard integrals to know the area or the length of a curve but were not able to answer the question what the geometric sense of the roots of the quadratic equation $x^2 - 2Hx + K = 0$ is, where H and K are the average and the Gaussian curvature of a surface. However, $H = k_1 + k_2$ and $K = k_1 k_2$ by definition, where k_1 and k_2 are the principal curvatures of the surface, and the students had known these formulas. So I taught them the quadratic equations at their final exam.

Galperin was astonished when his students told him that he taught them "Russian mathematics". In fact, he taught them just mathematics rather than so-called new-new math.

[2] Senator Byrd's speech deserves careful reading. You can find it at **http://www.intres.com/math/byrd.htm**.

Another example: When I was in school, we studied geometry as a deductive system and proved theorems. Although our study was imperfect, the idea was stimulating. In American education this may not even be considered desirable. How do I know this? In the last twenty years the National Council of Teachers of Mathematics (NCTM), a very powerful organization, published three documents expressing its vision of mathematical education: "Agenda for Action" [1] (1980), so-called "standards" [4] (1989) and "Principles and Standards of School Mathematics" or PSSM for short [17] (2000). Since PSSM is not intended to substitute "standards" and treats "standards" quite respectfully, criticism of "standards" remains necessary. The two last volumes of the three-volumed "standards" are almost never discussed, so little mathematics they contain. We shall speak only about "curriculum standards", to which we shall hence refer to as "standards" and which constitute the greater part of the first volume [4]. "Curriculum standards" consists of three parts pertaining to the elementary, middle and high school and we shall concentrate our attention on the two latter parts. I have never taught at the elementary-school level and shall not comment on the elementary-school part of "standards". I only want to say that the elementary-school part seems to be more reasonable than the other two parts. For example, it recommends to increase attention to "mental computation" and "word problems with a variety of structures" (p. 20) with both of which I wholeheartly agree. I would be happy to see similar recommendations in the other two parts, but they are not there.

First of all, I must say that "standards" is a difficult reading for a mathematician who has got used to expect exact meanings. It is written in a very fancyful manner, many words have strange meanings or seem to have no definite meanings at all. For example, chapters are called "standards", which gives impression that there are some standards there. [3] But if you apply effort and concentrate, you notice that this looseness is not only in *how* it is written, but also in what it *recommends.* This document is written by people, for whom all mathematics is but a disordered collection of interchangeable appendices to their vague generalities. (The same is true of PSSM.) The "standards" contain several reasonable problems (along with several unreasonable ones), but all of them are torn out of their natural mathematical context We, mathematicians, have got used that mathematics is structured around its content and that all its statements are connected, and that the logical inference is the most important connection. The "standards" carefully avoid to speak about logical connections. Each part contains a chapter named "Mathematics as reasoning", but all the three chapters contain very little reasoning. Through its long history mathematics has collected many important but elementary proofs to fill a chapter with such a title, for example many beautiful geometrical theorems including the famous Pythagorean theorem.

[3] Chapters of PSSM are also called "standards", which leads to the same confusion.

Also the famous proofs that $\sqrt{2}$ is irrational, that when a rational number is turned into a decimal fraction, it is periodic and vice versa and that there are infinitely many prime numbers. I would also put there criteria of divisibility by 3 and 9 (and 11) and some of the most elegant proofs by the method of mathematical induction and some of the most thrilling fallacies and paradoxes. As a university teacher, I certainly want my students to understand all this and solve related problems. Is it possible to study proofs in high school? Theoretically, yes. We are not aware of any biological or psychological law to forbid this. According to Piaget, most normal children reach the stage of formal operations (the last stage in his theory) around 12 years old (plus-minus a few years). More than that, we have proof by experience that this is possible: some schools successfully teach such things [20]. Remember also mathematical olympiads where teenagers solve problems which need very rigorous reasoning. However, this theoretical possibility is realized in practice only when there are favorable conditions, first of all good teachers who know and love their subject. [4]

Now about the three chapters "Mathematics as reasoning" in the "standards": none of the famous facts listed above is there. What is there? The high-school chapter starts with tampering with calculator. If you don't believe me, look by yourself. What about the Pythagorean theorem, it is mentioned in the "standards" on pp. 113-114 with a well-known picture, which can be used to prove it, but it is only proposed to use this picture to "discover this relationship through exploration". The possibility to prove this important theorem is not even mentioned and the very idea of proof is avoided throughout the document.

In PSSM I first could not find a mention of Pythagorean theorem at all although I looked for it in the sections "Geometry"'and "Reasining and Proof". Then I found it in the section "Algebra". The book has no index and to critisize it one has to start with making an index by hand; this is too time consuming, so the book remains uncriticised.

The authors of "standards" think that they write about mathematics, but all they write is "out of focus", like a bad photo. They start with some generalities, some of which may seem reasonable at first, but do not especially care which concrete mathematical content (if any) they use to illustrate them. This makes a sharp contrast with Russian programs which contained no vague generalities at all, just detailed lists of topics, which were very difficult to misinterpret. In those few cases when vague generalities were included into Russian programs, they always led to negative consequences. For example, a program in physics

[4] To provide appropriate conditions for them is also very important. This needs collaboration and proper sharing of functions between various people including students and teachers themselves, mathematicians, parents, educational administrators, politicians and general public.

for university entrance examinations recommended to distinguish between “genuine understanding” and “having been coached”, which immediately moved some examiners to frown upon applicants who knew too much and therefore were perceived as “coached”. Generally I am convinced that there is nothing worse than include words like “understanding” into official documents. Modern American educators do exactly this: PSSM abound in recommendations to achieve “deep understanding” but fail to specify what exactly should be understood.

Another unhappy feature of “standards” is complete absence of connections with other sciences. Isolation of subjects from each other, notably between mathematics and physics, is a chronic disease of American education. What about physics, many American school students simply never take it and nobody tells them that they miss something important. Once, teaching a course of calculus, I solved a mechanical problem and then said: “The same result can be obtained from the law of conservation of energy.” Silence. I asked: “Who has ever heard about the law of conservation of energy?” There was one foreign student in the group (from Taiwan) and he was the only one to raise his hand. In Russia every school student was required to take all the main courses including mathematics and physics and the courses of mathematics and physics were always stronly correlated. The course of physics was full of problems that needed algebra, geometry and trigonometry to solve and the course of mathematics included many problems with physical content.

It must be said that the “standards” have a wide popularity among educators. I think it is because the vague feelings of the authors are close to the vague feelings of their audience. All of them feel that their teaching is too rigid, mechanical, uninspired and want to make it more flexible, more human, but they are not competent enough to keep mathematics on this way: as they move towards more human approach, they lose mathematics.

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When American educators speak about mathematical education in Russia, they usually remember some special projects, such as mathematical schools [20] and circles [6]. (See also my review [24].) Indeed, all of them were and are useful, but I want to say something else. When I taught mathematical circles where we solved non-standard problems, I took for granted my students' basic knowledge and skills and ability to solve standard problems. Without all this we, university students, unexperienced in teaching, would not be able to teach advanced topics.

When I came to America, I taught several classes of problem solving and started to appreciate much more the basic education, because my new students dramatically lacked it. They understood advanced ideas but floundered in algebraic transformations, which turned solution of interesting problems into painful struggle with basics. For example, when we studied the method of mathematical induction, my American students understood the idea (which is not trivial), but got into trouble when they needed to substitute $n+1$ instead of n into a formula. This showed to me that there was something wrong with the most basic level of mathematical education in America. In this I am not alone as you will se below.

In Russia I had another useful experience: collaboration in the School by Correspondence, most students of which lived in villages and small towns and were culturally deprived by comparison with children of metropolitan areas. In addition, we could not meet with them face to face. We sent them brochures which included problems, the students solved them as they could and sent their solutions to us. We checked and commented their solutions and sent our comments to them, after which the students could try to solve the same problems again. For example, the following problem was used: "Is there a triangle, whose area is greater than 100 square meters, but every height is less than one centimetre?" This problem is useful for at least two reasons: First, it moves students to expand their supply of particular triangles with various properties. Second, it helps to distinguish between a vague idea and an exact description. Some students replied: "Yes, there is such a triangle. It should have a very large base and a height equal to only a few millimeters." Such a bizarre amalgama of insight and confusion is typical whenever students solve problems on the border of their possibilities and a major problem in mathematical education is how to react to it. On one hand, the student who hit at this idea should be encouraged, because it is a valuable insight, but on the other hand it would be a very bad service to that student not to make him aware how much this insight is short of a mathematically correct solution. On request of the School by Correspondence I wrote instructions for its teachers. In this case I recommended to answer in the following vein: "What you wrote is a valuable *draft*, which may lead to a solution, but it is not yet a solution. Now, based on your draft, describe some concrete triangle exactly and show that its area and heights satisfy the required inequalities. Only having done this you will solve the problem!" Problems of this sort do not yet involve explicit proofs, but they develop mental discipline which is necessary to deal with proofs. It seems to me that such preparations are rare in USA. The "standards" never mention mental discipline and the very phrase "mental discipline" has turned into a pejorative term among American educators. What about the "standards", its idea of advanced topics can be illustrated by the excursus into the fractal geometry (p. 157), which, of course, is quite superficial. (The very word "dimension" is not mentioned at all.) The idea to teach fractals in school has already found many supporters. (Everything is possible for those who are not competent enough to understand how difficult it is.) I asked several school teachers who were enthusiastic about teaching fractals to define a fractal and none of them mentioned the idea of dimension, least defined it. Usually they emphasized "repeating patterns". When I asked why they were not satisfied with wall-paper, they took offence.

Why fractals? I think, it is because for many years American educators have been accused of "dumbing down" their students and now they desperately try to show that they care about some advanced topics, but some of them are not competent enough to choose these advanced topics realistically.

Another example of this: on p. 157 the "standards" recommend to "develop an understanding of an axiomatic system through investigating and comparing various geometries", which is widely interpreted as a suggestion to teach non-Euclidean geometry, which is impossible to do having eliminated almost all the logical structure.

Still another example:

> College-intending students should become familiar with such distributions as the normal, Student's t, Poisson, and chi square. Students should be able to determine when it is appropriate to use these distributions in statistical analysis (e,g., to obtain confidence intervals or to test hypotheses). Instrictional activities should focus on the logic behind the process in addition to the "test" itself. [4, p. 169].

Do the authors know that the theory of continuous random variables, where normal, Student's and chi square distributions belong, needs such an amount of calculus as a prerequisite, which usually takes three semesters? Do they know that calculus also needs certain prerequisites, which take years to teach, but are too often neglected in America? (According to TIMSS, among all students in the world who take calculus in school, American students are very low in knowledge of pre-calculus topics.) The authors of "standards" want to reform American mathematical education, but actually only aggravate its main shortcoming: vain ambitions and contempt for consistent, systematic and thorough study. Absence of any organized curriculum made neglect for prerequisites a chronic disease of American education.

Also, what do the authors mean by "becoming familiar"? It seems to be something quite superficial. Nowadays all American children are familiar with the ideas of cosmic travel, travel in time and robots because such films as "Star Trek", "Terminator" and "Robocop" brought these ideas into every home.

This is not bad at all, but this does not make children capable to operate space ships or design robots. It is necessary to distinguish the useful, but superficial level of familiarity, provided by the entertainment industry, from the mastery and understanding which should be achieved in school. Are the authors of "standards" aware of this difference?

TIMSS followed American educators in their neglect of theory (but not in their bombastic ambitions; it contained no problems on fractals, non-Euclidean geometry or Student's or chi-square distributions). [5] Algebra and geometry, the two academic subjects most appropriate for study at school, constituted only a smaller part of its "literacy" part and what was presented as "algebra" might be very far from it. For example, the following problem was included into the middle-school part of TIMSS as item 13 and classified as algebra [12, p. 76]:

> These shapes are arranged in a pattern:
>
> ○△○ ○△△○ ○ ○△△△
>
> Which set of shapes is arranged in the same pattern?
>
> | | |
> |---|---|
> | *A* | ⋆□⋆□⋆⋆□□⋆⋆□□ |
> | *B* | □⋆□□⋆□□□⋆□□□□ |
> | *C* | ⋆□⋆⋆□□⋆⋆⋆□□□ |
> | *D* | □□⋆⋆□⋆□□⋆⋆□⋆ |

Even if it made sense to include this puzzle into TIMSS (about which I am not sure), it is not even close to algebra! The fact that they call it algebra shows how little of real algebra is there. We cannot even reproach the organizers of TIMSS for their neglect of theory. If TIMSS included more mathematics and less fads, some educators would be still more eager to dismiss its results.

Even the *advanced* part of TIMSS includes very few theoretical topics or problems on proofs: of all its 36 items released on the web [6] only one (K-18) requests to write a proof. Most of the other items require just to choose an answer in a multiple choice format and/or to obtain a numerical answer. Some items contain a requirement "show your work", but the work to show is mostly numerical computations. Even algebraic transformations are almost absent.

[5] PSSM also dropped these topics.

[6] **http://timss.bc.edu/TIMSS1/items.html**

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What was the position of American mathematicians towards the "standards"? This is another mystery. The preface to the "standards" declares (p. vi): "The following mathematical science organizations join with the National Council of Teachers of Mathematics in promoting the vision of school mathematics described in the Curriculum and Evaluation Standards for School Mathematics", followed by an impressive list including the American Mathematical Society (AMS). What does this "promoting the vision" mean? Neither AMS nor NCTM ever made a public statement about it. When the "standards" started to be implemented in some classrooms and mathematicians became aware of what was going on there, they became horrified and only then, probably, some of them looked attentively under the cover of "standards". Notices of AMS published several letters urging AMS to withdraw its endorsement (whatever it meant), but there was no comment from the headquarters of AMS.

Since the "standards" seemed to be endorsed by so many highly scholar organizations, it is no wonder that many teachers declared that they teach according to these "standards". When these teachers were asked why did they think so, most of them answered: "Because my students use calculators instead of doing paper-and-pencil calculations". This was said with pride because the "standards" really abound in hints to increase attention to use of technology, including calculators, and to decrease attention to paper-and-pencil calculations. There were bombastic promises that this would release children's time to acquire "high-order thinking skills", but in fact the opposite was observed. Many university teachers complained that their students cannot do simple calculations. The following was sent to an e-mail list by Lawrence Braden, a well-known teacher, one of the authors of [18]:

> The 'do not teach the child fractions except by calculator' is not a rare thing these days. What would be condemned as heresy twenty years ago is now accepted orthodoxy in many circles. One state prides itself in *not* requiring students to know how to add one third to one seventh by hand, or to be able to multiply two two-digit numbers together by pencil and paper, or to be able to divide 10 by 1.05 without a calculator. Such drills are deadly dull, and the time must be spent instead to foster 'higher-order thinking skills'. I am not making up these examples; they were actually used in front of a roomful of witnesses this summer. I took notes. Those subject to bad teaching in years past (Just invert and multiply, kid, that's how we divide fractions, just *do* it) at least could *do* it. The kids today not only cannot do it, they cannot do anything *else* either. Except perhaps to invert matrices and find the "best-fit" line on a calculator.

My family came to USA soon after the "standards" was published. At first I knew very little about the so-called "reform" of the mathematical education in America, but I observed that my daughter' teacher of mathematics foisted a calculator into her hand all the time. I understood that I had to do something radical and started to call my daughter "a victim of American education" whenever I saw a calculator in her hand. By the end of high school she was one of a few students who could calculate mentally. Many university freshmen grasped a calculator when they needed to calculate ten percent of a number. Sometimes I tore calculator from a student's hand shouting: "You can do it without a calculator!" The student gazed at me for a while in astonishment, then realized that he really could, but I was the first person who cared to tell him that it made sense. The "standards" turned usage of calculators into a matter of prestige and some teachers started to feel obsolete and inadequate without them.

Then a really dramatic event followed. In October of 1999 the US Department of Education headed by Richard W. Riley approved ten K-12 mathematics programs by calling five of them "exemplary" and other five "promising". [7] This decision was based on conclusions of an Expert Panel, most members of which have neved published a research article in mathematics. [8] This is not an accident. Although there are plenty of bright mathematicians in USA, for a long time they were not invited to participate in making important decisions about education. Sincerely speaking, they did not especially object. Like all people, mathematicians are prone to avoid extra work and sometimes say: "Why should I bother myself with public education? There are special people to care about it." So they did for a long time in America (but not in Russia). However, this time some of them decided to act. On November 18, 1999 The Washington Post published a letter signed by 200 mathematicians and other scientists urging Riley to withdraw his department's approval. NCTM immediately expressed a complete support for Riley's decision [9], which is understandable because the Expert Panel based its criteria partially on the "standards". Riley answered to the mathematicians' letter by reaffirming his position [10]. Thus we observe an open confrontation between mathematicians and scientists on one side and educational officials and leaders on the other. What is it about?

The most evident point of confrontation is whether children should be taught paper-and-pencil arithmetical algorithms or use calculators instead of that. The difference of opinions can be illustrated by two quotes, both included into the mathematicians' letter. One is from an article written by Steve Lein-

[7] The approved programs are listed and described at **http://www.enc.org/ed/exemplary/**.

[8] The list of members of the Expert Panel is available at **http://www.ed.gov/offices/OERI/ORAD/KAD/expert_panel/mathmemb.html**.

[9] **http://www.nctm.org/rileystatement.htm**

[10] **http://www.ed.gov/News/Letters/**

wand, a member of the Expert Panel (and one of directors of NCTM), entitled "It's Time To Abandon Computational Algorithms" and published on February 9, 1994, in Education Week on the Web: [11]

> It's time to recognize that, for many students, real mathematical power, on the one hand, and facility with multidigit, pencil-and-paper computational algorithms, on the other, are mutually exclusive. In fact, it's time to acknowledge that continuing to teach these skills to our students is not only unnecessary, but counterproductive and downright dangerous.

This is just a personal opinion, but this person was selected by the government to make an important decision about mathematical education.

The other quote is from a report made by a committee formed by AMS for the purpose of representing its views to NCTM:

> We would like to emphasize that the standard algorithms of arithmetic are more than just 'ways to get the answer' – that is, they have theoretical as well as practical significance. For one thing, all the algorithms of arithmetic are preparatory for algebra, since there are (again, not by accident, but by virtue of the construction of the decimal system) strong analogies between arithmetic of ordinary numbers and arithmetic of polynomials. [12]

Pay attention that this statement was made only in 1997 and published only in 1998, which was too late because the "standards" had recommended the usage of calculators instead of paper and pencil already in 1989 and at that time AMS seemed to support it.

In Russia mental and pencil-and-paper computations were always recommended throughout the school and considered essential for understanding. For example, Igor Arnold wrote in his book "Logarithms in the school course of algebra" (I don't have this book with me and quote from memory): "We tell students that $\log_{10} 2 \approx 0.30103$ because one can obtains 2 by raising 10 to this degree, but the problem is that the student has never seen anybody 'obtain' 2 in this way." In view of this, Arnold recommended to teach students to estimate logarithms by mental calculations and by hand, without tables. (Calculators were not available in the 30s, when Arnold wrote his book, but it is clear that he would say the same if they were.) For example, $2^{10} = 1024$, which is a little more than 10^3, whence $\log_{10} 2$ is a little more

[11] **http://www.edweek.org/ew/1994/20lein.h13**

[12] American Mathematical Society NCTM2000 Association Research Group Second Report. June 1997. Notices of the AMS, February 1998, p. 275. See **http://www.ams.org/notices/199802/comm-amsarg.pdf**.

than 0.3. Hence $\log_{10} 5$ is a little less than 0.7, $\log_{10} 4$ is a little more than 0.6 and $\log_{10} 8$ is a little more than 0.9. After that, using interpolation, we can estimate that $\log_{10} 9$ is a little more than 0.95, whence $\log_{10} 3 \approx 0.48$, whence $\log_{10} 6 \approx 0.78$. Interpolation between 6 and 8 gives $\log_{10} 7 \approx 0.84$. There are many other numerical relations which allow to check and improve these estimations. I believe that mental and by hand estimations are very useful in all areas of mathematics, including trigonometry and study of functions in general, and that they are essential as a preparation for calculus.

* * *

Thus "reformers" of mathematical education in USA have already excluded most theory and now want to exclude arithmetical algorithms from curriculum. What for? For the sake of problem solving. American educators have said many times that they care very much about problem solving. "Agenda for Action", which expressed the NCTM's vision twenty years ago, suggested that "problem solving be the focus of school mathematics $< \cdots >$ " [1, p. 1]. The "standards" completely share this opinion (p. 6) and start every part with a chapter called "Mathematics as problem solving". Every section of PSSM also contains a chapter named "Problem Solving". [13] In my opinion, solving problems is really very important, I even suggest that public mathematical education **IS** teaching children to solve mathematical problems, provided the words "problem" and "to solve" are interpreted properly. After all, any mathematical theory can be represented as a series of problems. So my first reaction to these declarations was positive. Suspicion came to me only when I noticed that every time when these educators declare their concern for problem solving, they try to exclude something from curriculum. If all their proposals are accepted, nothing remains but mirages. In this article I concentrate on three trends, which seem the most dangerous to me: to exclude theory for the sake of "hands-on" approach, to exclude arithmetical paper-and-pencil calculations for the sake of "high-level thinking skills" and to exclude traditional word problems for the sake of "real-world problems". We have already addressed the two former follies, now we are going to the third one.

The failure of New Math (see [8] for example) showed that we cannot teach children rigorous and abstract mathematics without a solid preparation. That is why we need problems on the boundary between mathematics and common language and common sense. There are many excellent problems of this sort and all of them are called *Word Problems* or *Verbal Problems* or *Story Problems*. Their

[13] What is ridiculous is that this chapter is there along with others, including "Algebra" and "Geometry" for example. So, according to the authors of PSSM, there is problem solving without any partilular topic and there are algebra and geometry without problem solving.

distinctive feature is usage of words, which are not mathematical terms, such as cars and trains, distance, time and speed, boats and current, planes and wind, boxes, cans and balls, length, widht and height, perimeter, area and volume, pipes, pumps and pools, mass, percentage and mixtures, hours, minutes and time of the day, hands of a clock, years and age, money, price, interest and discount, etc. Typically, a word problem describes some imaginary situation, provides certain data about it and asks a question, the answer to which can be obtained from these data. To solve it, students need to interpret the given situation mathematically. Thus every word problem is a small instance of mathematical modeling. In Russia presence, even abundance of word problems in mathematical education was always normal. "Always" does not mean only Soviet time; it was so already in the 19-th century. In fact, even the term "word problem" is not used in Russia, because the word "problem" usually means a word problem, while non-word problems are called "exercises". Also, word problems are not considered only a part of algebra. First, they are present much earlier, from the very beginning of elementary school, and are supposed to be solved without algebra. Second, they provide contacts of algebra with geometry and physics and generally the world of material objects. Let me give a few examples.

A problem for the 3-th grade (9 years old):

> A grocery shop has 400 kg of flour to pack. It has 85 bags which can contain 2 kg and 75 bags which can contain 3 kg of flour each. Are these bags sufficient to pack all the flour? [13, p. 177]

A problem for the 4-th grade (10 years old):

> A library needs to bind 4500 books. One shop can bind these books in 30 days, another shop can do it in 45 days. In how many days can the two shops fulfill this order working simultaneously? [14, p. 197]

Two problems for 5-6-th grades (11-12 years old):

> When milled, wheat loses 1/10 part of its weight. How much bread can be obtained from $1\frac{1}{2}$ ton of wheat if when bread is baked, the surplus weights 2/5 of the flour used? [3, p. 114]

> A boat, going downstream, made a distance between two ports in 6 hours and returned in 8 hours. How much time is needed for a raft to make this distance downstream? [3, p. 246]

Some problems for 6-8-th grades (12-14 years old):

> (An ancient problem.) A flying goose met a flock of geese in the air and said: "Hello, hundred geese!" The leader of the flock answered to him: "There is not a hundred of us. If there were as many of us as there are and as many more and half as many more and quarter as many more and you, goose, also flied with us, then there would be hundred of us." How many geese were there in the flock? [10, p. 37].

> (Historical problem.) A swimmer was swimming upstream Neva River. Near the Republican Bridge he lost an empty flask. After swimming 20 min more upstream he noticed his loss and swam back to find the flask; he reached it near the Leughtenant Schmidt Bridge. Find the velocity of current at Neva River if the distance between these two bridges is 2 km. [10, p. 208].

> A train started from a station and, moving with a constant acceleration, made a distance of 2.1 km and ended with a speed of 54 km/hour. Find the acceleration of the train and the time it spent. [10, p. 257].

> The rectangular lid of a box has length 30 cm and width 20 cm. A rectangular hole with the area 200 square cm must be made in this lid so that its sides were at equal distances from the sides of the lid. Which should be the distances of sides of the hole from the sides of the lid? [10, p. 259].

If American students were taught to solve such problems in a sytematic way, they would be prepared better than many of those who are actually sent to calculus in high schools and colleges. For example, a problem similar to the boat problem quoted above was included into my article [23]. After its publication I received many letters from which it was clear that this problem was difficult even for university students.

It is common for Russians, both mathematicians and non-mathematicians, to remember word problems with sympathy. For example, Vladimir Arnold, a famous mathematician (son of Igor Arnold quoted above), remembers warmly how he solved a difficult word problem in high school [2]. This is understandable. Study of mathematics in school is useful because it teaches children to understand complex, rigorous or abstract matters. When we teach children to solve problems in school, we do not expect them to meet exactly and literally the same problems in later life. Mathematical education would be almost useless if its only use were literal. We want much more, we want to teach children to solve problems in general. In this respect traditional word problems are especially valuable, because to solve a word problem, you have to *understand* what is said there. This function of word problems is very poorly understood in America. Although American educators pay lip service to the memory of George Polya, they often neglect his opinions. Polya wrote:

> *Why word problems ?* I hope that I shall shock a few people in asserting that the most important single task of mathematical instruction in the secondary schools is to teach the setting up of equations to solve word problems. Yet there is a strong argument in favor of this opinion. In solving a word problem by setting up equations, the student *translates* a real situation into mathematical terms; he has an opportunity to experience that mathematical concepts may be related to realities, but such relations must be carefully worked out. [16, p. 59].

When I read [16] in Russia, I thought that it was just a good book. Now I understand how polemical it actually was. Its first two chapters are devoted to two Cinderellas of American education: classical geometry and word problems. Polya loved traditional word problems. You think that this is natural? Read further.

In America word problems (also known as story problems or verbal problems) are not taken as easy as in Russia, They arouse mixed, even contradictory feelings. On one hand, they are considered especially difficult, even frustrating. Generally, frustration caused by mathematics is one of the most noticeable concerns of American educators, which is understandable because schools of USA have no established curriculum, so very often there is no continuity in the courses a student takes: some courses repeat each other, some are separated by gaps; in the former case boredom and in the latter case frustration is inevitable. Word problems seem to create the greatest frustration. I think, it is because their difficulty is not so evident, a good deal of it is in syntax and semantics of the natural language rather than in a well-established, recognizable mathematical theory, so in their case American administrators are especially careless about continuity. There is a cartoon in the Far Side series called "Hell's Library" showing a library full of story problem books. Mildred Johnson, an experienced teacher, starts her book about word problems (actually very easy ones against Russian standards) as follows: "There is no area in algebra which causes students as much difficulty as word problems." [7] Pay attention that Johnson does not mention arithmetical word problems, which are unknown in USA.

During some time I participated in an e-mail list devoted to teaching of mathematics. Most of it is on the web, so you can read messages sent to it. [14] Also you can search it for key words and phrases. You will find several relevant messages searching it for "word problem frustration". In particular, under the subject line "speed of a current river", you can find a discussion of a problem similar to the Neva River problem quoted above. In Russia this problem was always considered elegant and interesting, but in America it was otherwise. One participant observed that this problem caused too much frustration and proposed to solve some other problems instead of it. I expressed my astonishment and then Don Coleman, an experienced teacher, replied: "Andre, $< \cdots >$ it is perfectly clear that such problems cause frustration. It is not a matter of explaining why it is. It just happens."

On August 20, 1996 I sent to this list several messages previously collected by me, all written by competent teachers. These are a few quotes from them: Eric L. Green:

> Somebody asked why word problems were so rare in math textbooks. The reason should be obvious – they scare elementary school teachers to death. Why do you think we had the discussion on 'keywords'? That's just one device elementary school teachers use to 'insure success', i.e. eliminate the need for kids to struggle and think. $< \cdots >$ I've noticed that many high school teachers avoid word problems for the same reason – to avoid frustration. In this case, kids have been trained for years to view math as a set of algorithms for solving particular problems.

Lynn Nordstrom:

> As an elementary teacher who works with many other elementary and middle school teachers, I agree with Eric when he says word problems 'scare teachers to death' and that provide a lot of frustration for all students. In my discussions with teachers, I find that many of them feel this way because they feel unprepared to teach mathematics.

Mark Priniski:

> Oooo... Word Problems! Why aren't there more of them in the text? Here's a story from the

[14] To find messages of this list sorted in the chronological order go to **http://forum.swarthmore.edu/epigone/math-teach/all**.

> past... The first year I taught Algebra (17 years ago) I was approached by my department head. He told me that the rest of the math department just skipped the word problems because they were too difficult for the students. (Some of my students would like me to take his advice :))"

It may look like a contradiction, but at the same time word problems have a reputation of being boring and trivial. The following statements were also made at e-mail lists. Mark Saul, a famous teacher, wrote:

> In New York State we have Regents exams, which kids must take at the end of certain courses. For years, these problems were staples on the exams. To get good grades, and so that the teacher could look good, there were review books published, which gave specific methods for solving these problems. For example, in an upstream-downstream problem, you have a 2x3 array of boxes (distance, rate, time across the top; upstream and downstream along the side). All the students had to do was memorize the labels for the boxes $< \cdots >$ "

Ralph A. Raimi, one of not so many professional mathematicians really interested in education, one of the authors of [18], remembers:

> In 1937 I was in the 9th grade and learned word problems, "story problems" as they were called. My teacher hadn't the foggiest notion that mathematics was written or thought about in English, and believed that story problems were like French regular verbs. They came in four types, each with its own endings $< \cdots >$ "

Judith Roitman, a well-known educator, one of the authors of PSSM, remembered:

> The high school algebra course I took (honors, too) was nothing but imposed charts and imposed algorithms. It was boring boring boring and if anyone had told me I'd grow up to be a mathematician I'd have laughed at them.

I think that the reason of this seeming contradiction is that word problems have an enormous potential of variability, and exactly for this reason they are often reduced to a few types which can be solved mechanically. This is done in America, but not in Russia, where the *quantity* of word problems to solve is simply too large to reduce them to types, so that nobody ever tried to do this. Many American students are so unprepared to solve word problems that even a small unexpected twist makes these problems unsolvable for them. My undergraduate students insisted that every problem on the test should be preceded by solving the same problem in class, only with different numbers (see [25]). Even change of a sign made a problem different for them, for example when a train moved in the opposite direction or a pipe drained water from a pool instead of bringing it there. Thus, to keep problems solvable, educators had to reduce them to a few types, everyone of which could be solved using a standard method and this is what the quoted authors observed. Their observations helped me to understand Agenda for Action's suggestion that " The definition of problem solving should not be limited to the conventional 'word problem' mode" [1, p. 3], but this suggestion was so inarticulate that no meaningful action could be undertaken based on it. The "standards" recommend to decrease attention to "word problems by type, such as coin, digit, and work" (p. 127), but this recommendation is silly for at least two reasons. First, it implies that coin problems form a type, that is all of them are similar, which is wrong. There are many different coin problems based on quite different ideas. Second, a recommendation to "decrease attention" may lead only to exclusion of some problems from curriculum, which can make the remaining problems only still more uniform. The main problem is not with the word problems, but with the poor preparation of American teachers. Polya quoted one prospective teacher say, "The mathematics department offers us tough steak which we cannot chew and the school of education vapid soup with no meat in it".

Since word problems are difficult for some teachers, it would be natural to say: "It is regretful that we are weak in solving word problems. We should pay more attention to them in schools of education. Textbooks and exams should contain more varied word problems. Educational journals should publish articles instructing teachers how to teach word problems. Textbooks should place word problems in a reasonable order, starting from easy ones and gradually increasing their difficulty and reaching quite difficult ones at the end." All this was done in Russia to some extent for a long time, but it seems that nothing of this was done in USA. Instead, educational leaders tried to create impression that there was something wrong with word problems themselves. For example, "Mathematics Teacher" (the main American journal for high school teachers of mathematics) published an article written by Zalman Usiskin, an influential educator, where he wrote: "Algebra has so many real applications that traditional phony word problems are not needed." [28, p. 158, 159]. In addition, this statement is repeated in a box in large type (p. 158). The editor's preface says that opinions of this article are close to those expressed in the "standards". Why does Usiskin call traditional word problems phony? He quotes the problem

"A person has 20 coins in his pocket, some nickels and some dimes. The total value is $1.75. How many nickels and how many dimes were there?" and continues: "Since the coins were counted, shouldn't the counter have kept track of the number of dimes?" (p. 159). In Russia (and in most countries, as I believe) this strange argument would be ignored as a bad joke, only in America it is taken seriously. Follies mentioned above can only deprive children of competence in mathematics, but this one is much more dangerous: it ignores some of the most valuable features of human mind. This argument was originally proposed by Edward L. Thorndike, a well-known American behaviorist. One chapter of his book [22], called "Unreal and useless problems", applies this label to all those problems which cannot be faced in real life literally. Thorndike thought that such problems produce a sense of futility and proposed to exclude all of them from curriculum. My observations completely refute Thorndike's claim. For example, the Goose problem mentioned above is evidently fantastic, but it was liked by Russian students. I have no doubt that American children, like all children in the world, have fantasy and well may be interested in problems which Thorndike classifies as "unreal and useless".

Thorndike's pedagogical ideas were criticized very sharply several times. In particular, Lev Vygotsky criticised Thorndike very thoroughly in his works [29, 30]. Vygotsky wrote: "As it is well-known, Thorndike, logically developing ideas, underlying his zoological experiments, came to a very definite theory of learning, which Koffka's book refutes quite thoroughly, thereby liberating us from the power of false and prejudiced ideas." [29, p. 284]. The word "zoological" shows Vygotsky's anger, with which I completely solidarize. Indeed, Thorndike's theory treats human beings as analogs of animals reacting only to concrete stimuli, for whom a slight deviation from reality makes a problem irrelevant. In fact, Koffka showed that even with respect to animals, Thorndike was wrong. He wrought [9, p. 169]:

> We have, therefore, in these cases a true *transfer of training;* for the animal employed a procedure which was successful under certain conditions after these conditions had been altered, he did so in a manner appropriate to the alteration.

Nevertheless, Thorndike's neglect of reality was gladly followed by many American educators. Diane Ravitch wrote in her excellent book: [19, p. 69] "Despite his critics, however, Thorndike's views continued to have enormous currency; he was, after all, a towering figure in his field. His claims were embedded in pedagogical textbooks, most especially his own, and were taught to generations of teachers and administrators."

Regretfully, Thorndike's influence is not only history; it is still a reality. Usiskin's article [28] essentially repeats his suggestions. I wrote about the fantastic ideas underlying these suggestions in more detail in [26, 27], so shall not repeat myself here. I leave it to the reader to figure out what remains of the world civilization if everything that needs imagination is eliminated, because this is what Thorndike and Usiskin's suggestion essentially means.

I shall not explain this important issue in more detail here. You may want to look at [26, 27]. Let me say only that abstract thinking of adults grows out of imagination of children and traditional textbook word problems enhance this process (if taught well). The vast majority of problems used in mathematical education at all levels rely on the fundamental human ability of imagination and thereby are not "real" in the literal sense so beloved by some American educators. The majority of valuable school problems are "unreal and useless" according to Torndike's book [22], "hopelessly artificial" according to the second chapter of Kline's book [8] and "phony" according to Usiskin's article [28].

Since Usiskin declared that "Algebra has so many real applications", we may expect enough of them in his program UCSMP for Grades 7-12 declared "promising" by the Expert Panel, but this is not the case. The "Mathematically Correct Algebra 1 Reviews" [15] rated this program the lowest in "quality and sufficiency of student work" and said about it: "there are far too few problems for each subtopic and they fail to cover the upper difficulty levels. The coverage of word problems is especially weak as there is no good introduction to writing equations with variables for unknowns, far too little practice on this, and no word problems beyond the easy level." Where are those "so many real applications"?

Let me emphasize that I don't imply that mathematics has no applications. I am not so silly. What I mean is that most word problems used in education at various levels are not applications and pretending that they are only misleads and disappoints the students. For example, if a problem speaks about pipes bringing water into a pool, it is *not* an application of algebra to pool management. It is algebra. If a problem asks in how many ways can a committee elect a chair, a secretary and a treasurer it is not an application of combinatorics to committee work. It is combinatorics. If educators cannot yet explain this important issue in professional terms, they at least should not confuse it with pejorative terms.

* * *

What do "reformers" of mathematical education propose positively? The high-school part of "standards" contains a list of topics to increase attention, where the first place is given to "the use of

[15] **http://mathematicallycorrect.com/algebra.htm**

real-world problems to motivate and apply theory" (p. 126). What is a "real-world problem" and how to distinguish it from a "problem by type" or a "phony traditional word problem"? This is a very hard question. First of all, there is only one problem explicitly called "real-world" in the whole volume of "standards". Here it is [4, p. 139]: "*Real-world problem situation.* In a two-player game, one point is awarded at each toss of a fair coin. The player who first attains n points wins a pizza. Players A and B commence play; however, the game is interrupted at a point at which A and B have unequal scores. How should the pizza be divided fairly? (The intuitive division, that A should receive an amount in proportion to A's score divided by the sum of A's score and B's score has been determined to be inequitable.)" This is followed by "Problem formulation": "Consider the situation with the following data: The winning score is $n = 10$; when the interruption occurs, the score is A:B=8:7. The pizza will be divided in proportion to each player's probability of winning the game."

This "problem formulation" is equivalent to that described by Pascal in his letter to Fermat on August 24, 1654 and it is used in introductory textbooks of probability, e.g. in the excellent books by Kai Lai [5, p. 26-28] and Snell [21, p. 3-5]. Both Kai Lai and Snell refer to Pascal's letter and provide interesting historical background. On the other hand, the "standards" contains no references and completely omits all historical details, which makes all the situation look artificial. I asked many sympatizers of "standards" to refer to any of their acquaintances who ever played this game and received not one positive answer. If the author were serious about real-world fairness or equity, he or she might recommend to divide pizza equally or give a bigger piece to the more hungry player, but certainly *not* to divide food by gambling. Administrators of orphanages, boarding schools and summer camps would be horrified by this idea; they care that each pupil consumes all the food she needs for her health and not gambles it away. Further, what is the mathematical meaning of the word "inequitable"? If there is no such meaning, then how could it be "determined" that some division is "inequitable"? What does the word "determined" mean? The authors try to push the reader towards the well-known solution without explaining in which sense this solution is correct, which is anti-mathematical. Further, the authors pretend that the requirement to divide the pizza in proportion to each player's probability of winning the game appears only in the "Problem formulation", but in fact it is implicitly present from the very beginning, because otherwise it can not be "determined" that to share the pizza in proportion of the players' scores is "inequitable". This assumption is implicit, but unstated, which is anti-pedagogical.

Another illustration of how confused is the question of "real-world problems": let us compare the recommendation to increase attention to "real-world problems" (p. 126) with the recommendation to decrease attention to "word problems by type, such as coin, digit, and work" (p. 127). Since it is impossible to increase and decrease attention to one and the same thing, we have to conclude that coin, digit and work problems are not real-world and therefore coin, digit and work do not exist in the real world. This conclusion seems ridiculous, but it is a fact that after publication of "standards" some educators decreased attention to most traditional types of problems and increased attention to pizza problems. This practice was laughed at, but nobody explained what should be done instead.

Another bizarre consequence of the same recommendation: some people guessed that "real-world problems" are those which mention brand names. Some textbooks included problems like this: "The best-selling packaged cookie in the world is the Oreo cookie. The diameter of an Oreo cookie is 1.75 inches. Express the diameter of an Oreo cookie as a fraction in simplest form." [16] This produced a wave of criticisms, to which the publishing house representative Jack Witmer answered: "Time and again, teachers tell us that the use of real-world examples is effective in engaging students' interest and in enhancing the learning process." [17] This sounds unconvincing, but what to do instead? Nobody knows.

So it makes sense to ask: do "real-world problems" exist in reality or only in the imagination of professors of education? Browsing through "standards", I found two statements about these mysterious critters. On p. 76 (middle-school part) it is said:

> The nonroutine problem situations envisioned in these standards are much broader in scope and substance than isolated puzzle problems. They are also very different from traditional word problems, which provide contexts for using particular formulas or algorithms but do not offer opportunities for true problem solving.

The traditional word problems do not offer opportunities for true problem solving??? What a nonsense! What a dirty ignorance! With their narrow experience the authors pretend to set standards! These are true victims of American education! Are they aware of the rich resourses of excellent word problems around the world? What do they propose? Read further:

> Real-world problems are not ready-made exercises with easily processed procedures and numbers. Situations that allow students to experience problems with "messy" numbers or too much or not enough informations or that have multiple solutions, each with different conse-

[16] "Math textbooks salted with brand names raises new alarm" by Constance L. Hays. New York Times, March 21, 1999.
[17] **http://www.nytimes.com/99/03/28/letters/lwit.html**.

> quences, will better prepare them to solve problems they are likely to encounter in their daily lives.

This is silly. The very idea of school is that it is *organized* so that children do not waste their time: they are taught according to a carefully prepared curriculum and solve carefully designed, selected and edited problems. All waste, redundancy, irrelevancy and boredom, which abound in the real world, should be excluded from school. The authors want to abolish this efficiency and present this as an achivement!

Further, having described some properties of the "real-world" problems, it would be most natural to present several examples of them right after that, but it is not done. This problem immediately follows the text quoted above:

> Maria used her calculator to explore this problem: Select five digits to form a two-digit and a three-digit number so that their product is the largest possible. Then find the arrangement that gives the smallest product.

This problem is not useless, but it has **none** of the qualities mentioned above: there is neither too much nor not enough information and no multiple solutions. What are consequences of solutions of this problem? grades? Are the numbers involved "messy"? Nobody knows because nobody knows how to discriminate a messy from a non-messy number. This problem cannot be just "encountered" in daily life, it is intentionally invented to use in school. The pizza problem discussed above [4, p. 139] (the only problem in the book explicitly called "real-world") also has none of the properties attributed to "real-world problems" on p. 76.

Let me emphasize that my criticism is not directed against problems which have more than one or none answer or problems with missing data or other special kinds of problems. All of them have their place in education and teachers may use them for special dramatic effects (see [25]). What I am against is an apology of irresponsibility, an idea that teachers, authors of textbooks and educational officials should not carefully prepare the learning process. They should, and every special situation should be planned in advance, like special effects in theater or circus.

In fact, the description of "real-world" problems quoted above explains the authors' inability to provide examples of them. Since these perishable fruits are not ready-made, they cannot be found in a book, because any book is ready-made. Although the "standards" is an exceptionally careless document, it was revised once before publication (according to its preface), so it cannot contain "real-world problems" because a problem revised at least once cannot be real-world any more by definition. Real-world problems just happen in the actual course of daily life, at least this is what I conclude from the "standards". What the authors lose from sight is that a mathematician, used to concentrate on abstractions, is not the best person to deal with such events. An experienced handyman or family doctor or life-saver or police officer would be much more heplful. It may be a good idea to teach children to cope with emergency situations, but it is not mathematics.

Another relevant statement can be found on page 157 of "standards" (high-school part):

> Prior to the work of the ancient Greeks (e.g. Thales, Pythagoras), geometric ideas were tied directly to the solution of real-world problems.

Now imagine that a government official in the Ancient Egypt measures areas of farms (on which taxes depend) and every time gets multiple solutions with different consequences. You bet, this official would be hastily removed, least he might provoke a rebellion. In addition, problems solved thousands years ago, when there was no theory, were "real-world problems". But page 126 recommends to use them to motivate and apply theory. How can problems posed and solved in the absense of theory motivate and apply theory? Don't ask, because you will never get an answer. Throughout all these years of bitter arguments about what this or that phrase of "standards" really meant, its authors never interfered with explanations. It looks like they wrote the "standards" in such a somnambulic state of mind that afterwards could not explain rationally what did they mean. Nevertheless PSSM write about the "standards": "Since their release, they have given focus, coherence, and new ideas to efforts to improve mathematics education [17, p. ix].

Meanwhile the educational war continues. On February 2, 2000 there was a hearing on "The Federal Role in K-12 Mathematics Reform" [18] where opposite opinions were presented in a very sharp manner. In particular, Jim Milgram, a mathematician, mentioned "dramatic drop in content knowledge that we have been seeing in the students coming to the universities in recent years". I believe that Milgram is right. If you live on the top of a building, you have to care about the whole building. The building of mathematics is structured: when I teach a university course, I need to be sure that my students were taught good traditional elementary mathematics all the way starting from first grades. If I don't care about all this, who will? It takes a professional to understand how important it is. That is why I believe that professional mathematicians should care about mathematical education at all levels,

[18]**http://www.house.gov/ed_workforce/hearings/166th/ecyf/fuzzymath2200/2200.htm**

including elementary, middle and high school. My main point is that mathematicians should be involved in mathematical education in a special capacity corresponding to their competence.

* * *

Of course, these short notes are not a complete picture of American mathematical education. If there were nothing else, USA would not be able to support its powerful industry. There are many good teachers who tend to concentrate in private or special (e.g. charter) schools, where "rrreformers" cannot dictate to them. In private communications, these teachers often express mistrust or outright contempt towards the new-new math phraseology, but they avoid to bring their opinions to public. However, there are clear indications to the better, some of which you can find using the Mathematically Correct web site [19]. Milgram and Norris' report [15] explains why California had to abandon the "new new math" and adopt a much more reasonable educational framework. Raimi and Braden's report [18] shows how poor in quality are "standards" written by some states. Liping Ma's book [11] shows how poorly are American elementary teachers of mathematics prepared by comparison with their Chinese colleagues although their official credits are much more impressive. Diane Ravitch's fundamental book [19] abundantly shows that follies mentioned in this article have deep roots in American education through the 20-th century.

I hope that pretty soon many American parents will say to teachers of their children: "Stop pretending to do the impossible, but do the possible by hell! Don't pretend to teach our children fractals, but teach them fractions properly! Don't pretend to teach them non-Euclidean geometry, but teach them Euclidean geometry with proofs! Don't pretend to teach them Student's and chi-square distributions, but teach them elementary algebra with problems so that they would not complain on tests "we did not solve such problems"! Don't pretend to teach them phantoms like "real-world problems" but teach them to apply elementary mathematics to physics and computer science!" This will put American education on a much better place in the world competition.

Having read this article, somebody may think that I connect poor education with democracy. I do not. What I do think is that democracy has many aspects and free elections of political leaders is just one of them. Quality of education is not determined by quality of political structure and can deviate from it for better or worse. Let me illustrate this idea by two examples. There are 2354 problems in [3], a few dozens of which contain Soviet political propaganda. This is an example: "How many years passed from

[19] **http://mathematicallycorrect.com/algebra.htm**

the French burgeois revolution till the Great October socialist revolution if the former took place in 1789?" (p. 17). Here the political bias is evident, but from the mathematical point of view the problem is fair: the correct answer is arguably $1917 - 1789 = 128$. Soviet leaders wanted such problems to indoctrinate Soviet ideology along with teaching mathematics, but only half of their wishes came true: students learned mathematics, but got rid of the Soviet rule. Some rushed to the West taking jobs from those who were raised on problems like this: "A national magazine surveyed teenagers to determine the number of hours of TV they watched each day. How many hours do you think the magazine reported?" [4, p. 79]. It may seem very human to use this problem in class: every student will be able to say something and nobody will be completely wrong, so nobody will be frustrated. But when these kids grow up, they will regret the years wasted on such shallow "problems".

What is democracy in education? Let me mention one important parameter of it: students should be allowed to study objective reality rather than fads of educational leaders or fancies of their teachers. The most important purpose of mathematical education, as I see it, is to bring the students to such a level at which everyone of them can say to the teacher: "Now I can decide what is true and what is false and don't need you to tell me this anymore." Regretfully, all the aspirations of "reformers" of American mathematical education go in the opposite direction. Mastery of algorithms makes students self-sufficient - get away from it, make them dependent on Texas Instruments. Logical proofs develop students' mental discipline - get away from them also. Traditional word problems allow to determine the right answer - get away from them also. What the "reformers" promote, that is open-ended problems, "real-world" problems with clouds of answers, activities instead of problems, create a fuzzy world, in which students always have to guess the teacher's mind and can not learn to discriminate between right and wrong by their own means. How does this combine with the traditional American values of integrity and independence? I believe, it does not and in the long run one will prevail and the other disappear.

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