BOOK REVIEW

Edited by ANDY LIU, University of Alberta.

Colorado Mathematical Olympiad: The First 10 Years and Further Explorations, by Alexander Soifer. Published by the Center for Excellence in Mathematical Education, Colorado Springs, 1994. ISBN 0-940263-03-3, softcover, 188+ pages, US$19.95. *Reviewed by Andrei Toom, Incarnate Word College, San Antonio, Texas.*

According to my knowledge, the first Mathematical Olympiad for high school students was organized in Hungary in the last decade of the 19th century. But the country where Mathematical Olympiads most flourished was Russia. Olympiads, informal classes (called "circles"), popular lectures — all this gave a high school student who lived in Moscow in the 30's, 50's, 60's, 70's or 80's rich opportunities for development and provided an excellent start for many future mathematicians (including Alexander and me, for example). The sequel was often frustrating; that is why many of them (us) emigrated to the West and became politically free. But then a sad thing happened: most of us found no way to continue here the same productive combination of research and creative teaching which was so formative for us in Russia and remains a hidden source of strength for so many of us.

Alexander Soifer is a lucky exception. He not only brought Olympiad seeds with him, but also planted and cultivated them, and now the Colorado Mathematical Olympiad is a 10-year reality. Of course, other people also played various vital parts in this story, and Soifer carefully describes their contributions (and settles some personal accounts) in his book. I shall not repeat his interesting Historical Notes; they deserve to be read verbatim.

Still the most important part of the book are the problems of the first ten Olympiads and their solutions.

I believe that the main function of Mathematical Olympiads is to seduce students into thinking. In other words, a good Olympiad problem presents such a special and rewarding experience that some youngsters (later we call them "talented") become addicted to thinking for the sake of having this exciting experience again and again. From this point of view it is not so important when you solve the problem: during the allocated time or on the next day or even later. You can call this experience insight or you can say that something dawned on you or that some God sent you good advice. Anyway, a good Olympiad problem has some intellectual surprise locked inside as in a puzzle box. For me one of the most rewarding experiences of reading Soifer's book came from the following problem.

> **Problem 1.5.** *(A. Soifer and S. Slobodnik).* Forty-one rooks are placed on a 10×10 chessboard. Prove that you can choose five of them that do not attack each other. (We say that two rooks "attack" each other if they are in the same row or column of the chessboard.)

I tried to solve this problem and invented a cumbersome argument, which involved consideration of several cases. The solution provided by the authors is rather long also. But the second solution, found by students, is really beautiful. I shall not rewrite it; read the book or invent it, but remember: it takes only eight lines in the book and you don't need to consider different cases.

Some Olympiad problems can serve as an excellent preparation for the study of professional mathematics. Let us consider two examples.

> **Problem 5.1.** You are given 80 coins. Seventy-nine of these coins are identical in weight, while one is a heavier counterfeit. Using only an equal arm balance, demonstrate a method for identifying the counterfeit coin using only 4 weighings.
> An equal arm balance is a device composed of two plates suspended from an arm. By placing a set of coins on each plate, one can determine which set of coins has the greater weight, but can not determine by how much.

This problem belongs to a well-known class which may be called "information theory for children". The maximal number of coins, for which the problem is solvable, is 3^k, where k is the number of weighings. Proof in one direction is based on the fact that every weighing has three possible outcomes. Proof in the other direction is an actual scheme of weighing, which is described in the book (and in many other sources).

There is a useful game in the same vein: I think of an integer number between 1 and 30. You may ask me any questions to which I shall answer only "yes" or "no". (So you should not ask "what is the number ?"!) Find out the number using only five questions. (Here, since there are only two possible answers, I should think of a number between 1 and 2^k, where k is the number of questions.) Games, pastimes and puzzles like this are very seminal and I am sure that they contribute a lot to intellectual development of children. Intellectual careers are successful if they start as informal games, leisurely pastimes or even family jokes.

Another example:

> **Problem 4.3.** Each square of a chessboard which is infinite in every direction contains a positive integer. The integer in each square equals the average of the four integers contained in the squares which lie directly above, below, left, and right of it. Show that every square of the board contains the same integer.

The main condition of this problem can be written in the form

$$F_{x,y} = \frac{1}{4}(F_{x-1,y} + F_{x+1,y} + F_{x,y-1} + F_{x,y+1}), \qquad (*)$$

where variables $F_{x,y}$ are defined for all $x, y \in \mathbf{Z}$. In Problem 4.3 these variables are positive integers, but we may consider a more general case where they are any real numbers. This is a discrete analog of the 2-dimensional Laplace equation

$$\frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} = 0.$$

The two equations have some important common properties, including versions of the maximum principle, whence the idea of how to solve Problem 4.3 comes: just consider the smallest number. I became aware of the discrete Laplace equation (∗) as a high school student when some mathematician (whose name I forget) gave a public lecture which he called "Dirichlet Problem". And the lecture was quite understandable!

Before a study of differential equations, it is most advisable to study their discrete analogs; but none of the American schools where I have taught differential equations cared about it. The American education reminds me of a speedway where most students drive as fast as they can straight ahead towards graduation. They have no time to enjoy wonderful landscapes. No charming side streets, no mysterious gardens. Only standard road signs in the form of quizzes and tests. Everything that can be omitted, is omitted. Whenever there is a chance to skip a course, students never miss it. [1]

Olympiads represent a different, even opposite approach. They concentrate on intellectual difficulties instead of avoiding them. A student who solves a problem like 5.1 or 4.3 is not yet eligible to get a grade in a Computer Science or PDE course, but she deeply understands something that many who have grades, do not. And when she will take these courses, she will understand still more. Who will be a better, more creative scientist or engineer?

Problems given at Olympiads do not need to be really new (although sometimes they are and Dr. Soifer has authored many of them); it is sufficient if they are new for the participants. Using this, Soifer used the rich source of problems provided by many years of the Olympiad activity in Russia. But nobody knows where and when these problems first originated. For example, Soifer attributes the following problem to Tabachnikov and me, because he found it in our book [2]:

> **Problem 10.2. Four Knights.** Four knights are placed on a 3×3 chessboard: two white knights in the upper corners, and two black ones in the lower corners. In one step we are allowed to move any knight in accordance with the chess rules to any empty square. (One knight's move is a result of first taking it two squares in the horizontal or vertical direction, and then moving it one square in the direction perpendicular to the first direction.) Is there a series of steps that ends up with the white knights in diagonally opposite corners, and the black knights in the other pair of opposite corners?

Since it was I who put this problem into [2], let me say what motivated me to do it. I was thinking how to turn "non-standard" problems into stan-

[1] Needless to say, the situation in Canadian schools isn't much better.—*Ed.*

dard ones, that is, how to teach students to solve non-standard problems on a regular basis. (Many wise people claimed that this is impossible.) One idea which came to my mind was to train students to translate a problem from one "language" to another, that is, to change the mode of presentation. In Problem 10.2 this means to draw a graph whose vertices are the nine squares of the board and whose edges correspond to possible moves. As soon as you do this, you see the situation. (You "see" it in the two senses: visualize and understand.) Mainly for the same reason Rubinstein included a similar problem in his book [1], pp. 15 and 204. I had found this problem (perhaps in another version) in some old Russian puzzle-book.

The role of problem solving has been debated in American education for many years. Some universities and colleges even offer special courses of "problem solving". (I wonder what the other courses are for?) Perhaps the greatest achievement of Olympiads for high school students is that they show that teenagers can solve non-standard mathematical problems. Without Olympiads people might think that this is impossible for some "natural" reasons. When I tell my students today that I solved in a public middle school many of the problems they solve in college, they do not react. Perhaps they do not believe me and imagine that Russia is populated mainly by bears and KGB officers. Or, perhaps, they think that Russia is so far away that laws of nature may be different there. If they had taken a Mathematical Olympiad when they were in high school, their opinions might be different.

Some educators believe that every problem students solve must have an immediate practical relevance. Olympiads show clearly how ridiculous this demand is. As a rule, Olympiad problems deal with some imaginary situation, which seems to be very far from practice. It is most practical to organize Olympiads and participate in them, but this practicality is of a higher nature than some students and educators can understand. I have no doubt that most participants of the Colorado Olympiad will become useful scientists, engineers and, I hope, educators, because we need educators who can solve problems, we need them very badly.

Every review must contain some criticism. Although I am quite fond of Alexander's activity, I think that too many of the problems in the Colorado Olympiads are chosen according to his personal taste. An Olympiad must give equal chances to all participants, even those whose tastes may be different from the taste of the organizer. Problems of the Moscow Olympiads were proposed by many people, whose tastes balanced each other. I hope that the same balance will be achieved in Colorado Springs.

About the book: except for a few misprints (e.g., inequalities on pages 150–151), it seems self-explanatory. I hope that it will be translated into many languages, because it is a useful example of a successful transfer of an important cultural phenomenon from one country to another.

References:

[1] Moshe F. Rubinstein, *Patterns of Problem Solving*, Prentice-Hall, Inc., 1975.

[2] S. L. Tabachnikov and A. L. Toom, *Didactic Games*, 1987 (in Russian).